Singular limit - from compressible to incompressible, MHD with non-conservative boundary conditions

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Abstract

The aim of the talk will be to present some theory on singular limits in thermodynamics of viscous fluids and how to rigorously obtain incompressible models from compressible ones in certain regimes. We consider a general compressible, viscous, heat and magnetically conducting fluid described by the compressible Naiver–Stokes–Fourier system coupled with induction equation. In particular, we do not assume conservative boundary conditions for the temperature and allow heating or cooling on the surface of the domain. We are interested in the mathematical analysis when the Mach, Froude, and Alfvén numbers are small, converging to zero at a specific rate. We give a rigorous mathematical justification that in the limit, in case of low stratification, one obtains a modified Oberbeck-Boussinesq-MHD system with a non-local term or a non-local boundary condition for the temperature deviation. Choosing a domain confined between parallel plates, one finds also that the flow is horizontal, and the magnetic field is perpendicular to it. The proof is based on the analysis of weak solutions to a primitive system and the relative entropy method. This is a recent joint work with Florian Oschmann and Piotr Gwiazda.

Keywords: Naiver–Stokes–Fourier system, MHD, low Mach number, low Alfvén number, Oberbeck–Boussinesq–MHD, relative entropy method.

References

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