Incompressible heat-conducting fluid with large flux: long time solutions.

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Abstract

We consider the incompressible Navier-Stokes equations coupled with the heat equation in a cylindrical domain Ω parallel to the x_3 -axis. On the lateral part of the cylinder we assume the slip boundary conditions for velocity v, so there is no flow through the side wall, but there is flow through the top and bottom, where we have possibly large inflow and outflow. Moreover, we assume homogeneous Neumann boundary condition for temperature θ , meaning thermal insulation. With smallness of the x_3 -derivative of initial velocity, initial temperature and the external force field and smallness of derivatives of the inflow and outflow, we prove long time estimates for v and θ in Sobolev spaces $W_2^{2,1}(\Omega^t), t \leq T$ with any finite T. Next, by Leray-Schauder fixed point theorem we show the existence of solutions to the problem in space $W_2^{2,1}(\Omega^t), t \leq T$. Thanks to the special geometry of domain Ω we are able to prove the existence of solutions with large magnitudes of initial velocity, temperature and inflow-outflow functions.

Keywords: incompressible Navier-Stokes equations, heat equation, inflow-outflow.