

Asymptotic behavior ($t \rightarrow +\infty$) of solutions to Navier-Stokes equations under Navier boundary conditions

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Abstract

We study the asymptotic behaviour of the solutions to Navier-Stokes unforced equations under Navier boundary conditions in a wide class of merely Lipschitz domains of physical interest that we call *sectors*. The main motivations come from the celebrated results by Foias-Saut [1] related to the long time behaviour of the solutions to Navier-Stokes equations under Dirichlet conditions.

Here the choice of the boundary conditions requires carefully considering the geometry of the domain Ω , due to the possible lack of the Poincaré inequality in presence of axial symmetries. In non-axially symmetric domains we show the validity of the Foias-Saut result about the limit at infinity of the Dirichlet quotient, in axially symmetric domains we provide two invariants of the flow which completely characterize the motion and we prove that the Foias-Saut result holds for initial data belonging to one of the invariants.

Finally we study the long-time behaviour of the Dirichlet quotient in particular domains, e.g. $\Omega = (0, \pi)^d$ with $d \in \{2, 3\}$, providing further results both from analytical and numerical point of view.

Keywords: Navier boundary conditions, long time behaviour.

References

- [1] C. Foias, J.C. Saut, *Asymptotic behavior, as $t \rightarrow +\infty$ of solutions of Navier-Stokes equations and nonlinear spectral manifolds*, Indiana Univ. Math. J. 33, 3, 459-477 (1984).