

Elliptic equations in Lipschitz and in $\mathcal{C}^1$ domains

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Abstract

We are interested here in questions related to the study of some elliptic equations in bounded Lipschitz or $\mathcal{C}^1$ domains with **Dirichlet** or **Neumann** boundary condition.

Problem 1. Laplace equation.

$$\Delta u = f \quad \text{in } \Omega \quad \text{and} \quad u = g \quad \text{on } \Gamma \quad (1)$$

We will give here some new results on the **traces** of non smooth functions, harmonic or non-harmonic. Using in particular the interpolation theory, we are going to study the questions of existence and **maximal regularity** of solutions in **fractional Sobolev** spaces or with **weights** associated with the **distance to the boundary**.

Problem 2. Non degenerate case.

$$-\operatorname{div}(a \nabla u) = f \quad \text{in } \Omega, \quad (2)$$

with **Dirichlet** or **Neumann** boundary condition. Here the scalar function a is such that $0 < a_* \leq a \leq a^*$. We will concentrate on the case of **generalized solutions** in $W^{1,p}(\Omega)$ with $1 < p < \infty$.

Problem 3. Degenerate case. We will finally consider the following problem:

$$-\operatorname{div}(\varrho^\alpha \nabla u) + k \frac{u}{\varrho^\beta} = f \quad \text{in } \Omega, \quad (3)$$

with or without boundary condition and where k is a non negative constant and α and β belong to the interval $[0, 1]$.

Keywords: Elliptic problems, Lipschitz and $\mathcal{C}^1$ domains, maximal regularity, traces, fractional and weighted Sobolev spaces

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