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On the occasion of the 60th Birthday of Professor Šárka Nečasová

Speaker and Mathematical Inspiration

About the Honouree

Prof. Šárka Nečasová has contributed decisively to the development of applied mathematics, inspiring generations of researchers and students. Her commitment to research and community engagement reflects a passion for knowledge and a deep humanistic vision of mathematics.

A Message of Recognition

This tribute takes place during the MATHAPP2025 conference, in recognition of a career of excellence and dedication. We thank her for her lasting impact on the mathematical sciences and on all of us who have had the privilege to learn from her.

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UMBC

Name	Title
Amrouche, Chérife	Elliptic equations in Lipschitz and in C^1 domains
Angeles-Bellido, Maria	Time decay for non-Newtonian-micropolar fluids
Bhandari, Kuntal	On a compressible MHD flow interacting with thermoelastic structure
Bodnár, Tomáš	A remark on the objectivity of stress tensor in mass diffusive compressible fluid model
Caggio, Matteo	High and low-Mach number regimes for capillary fluids
Deuring, Paul	Stokes resolvent with traction boundary conditions
Eiter, Thomas	Viscous flow around an obstacle with oscillating boundary
Falocchi, Alessio	Asymptotic behavior of solutions to Navier-Stokes equations under Navier boundary conditions
Fraunié, Philippe	Numerical simulation of instabilities as driving processes in the ocean surface
Gazzola, Filippo	The steady Navier-Stokes equations in a system of unbounded channels with sources and sinks
Hacker, Kilian	Derivation and analysis of a Stokes-transport system modeling thermoregulation in human skin
Hieber, Matthias	Analysis of compressible hydrostatic flows
Juodagalvytė, Rita	Regularity of very weak time-periodic Poiseuille-type solutions
Kalousek, Martin	Existence of strong solutions to a class of compressible non-Newtonian Navier-Stokes equations
Kaplický, Petr	Maximal regularity of Stokes problem with dynamic boundary condition - Hilbert setting
Kreml, Ondřej	On wild solutions to the compressible Euler system
Kumbhakar, Jiten	Some controllability results for 1D linearized compressible Navier-Stokes equations
Lancmanová, Anna	Automated design and simulation workflow for an axial blood pump
Li, Qianfeng	Inversely designing boundaries from observed shock fronts in two gas dynamic models
Liu, Yadong	A diffuse interface model of Fluid-Structure Interactions for blood flows and thrombus
Mácha, Václav	On time-periodic solutions to an interaction problem between compressible viscous fluids and viscoelastic beams
Majumdar, Apala	Continuum theories for liquid crystals and their applications
Muha, Boris	Weak solutions for fluid-structure interaction problems with three-dimensional structural displacements
Neuss-Radu, Maria	Effective interface laws for fluid flow and solute transport through thin reactive porous layers
Neustupa, Jiří	A necessary condition for the vanishing singularity in a suitable weak solution of the MHD equations
Oschmann, Florian	Darcy's law for inhomogeneous incompressible flows
Pileckas, Konstantin	Non-stationary Navier-Stokes equations in 2D power cusp domain
Pires, Marília	Numerical modeling and simulation of viscoelastic fluid flows: Challenges and approaches
Pokorný, Milan	Weak solutions for compressible viscoelastic fluid models in three space dimensions
Rencławowicz, Joanna	Incompressible heat-conducting fluid with large flux: long time solutions
Roy, Arnab	A conditional no-contact result of compressible fluids and elastic plates in 2D
Sequeira, Adélia	Computational modeling of blood rheology in cerebral aneurysm flow dynamics
Silvestre, Ana Leonor	On the stationary Navier-Stokes equations in distorted channels and pipes under the DDN boundary condition
Specovius, Maria	Boundary value problems for the Stokes equations in an infinite layer
Tang, Tong	Energy conservation for compressible fluid systems with Korteweg stress tensors
Trifunović, Srđan	Contact in fluid-plate interaction: formation and detachment
Valášek, Jan	Mathematical and numerical model of human phonation
Varnhorn, Werner	On Leray's structure theorem
Webster, Justin	Uniqueness of weak solutions in hyperbolic-parabolic systems with applications in poro-elasticity
Wróblewska-Kamińska, Aneta	Singular limit - from compressible to incompressible, MHD with non-conservative boundary conditions

Elliptic equations in Lipschitz and in $\mathcal{C}^1$ domains

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Abstract

We are interested here in questions related to the study of some elliptic equations in bounded Lipschitz or $\mathcal{C}^1$ domains with **Dirichlet** or **Neumann** boundary condition.

Problem 1. Laplace equation.

$$\Delta u = f \quad \text{in } \Omega \quad \text{and} \quad u = g \quad \text{on } \Gamma \tag{1}$$

We will give here some new results on the **traces** of non smooth functions, harmonic or non-harmonic. Using in particular the interpolation theory, we are going to study the questions of existence and **maximal regularity** of solutions in **fractional Sobolev** spaces or with **weights** associated with the **distance to the boundary**.

Problem 2. Non degenerate case.

$$-\text{div}\,(a\nabla u) = f \quad \text{in } \Omega, \tag{2}$$

with **Dirichlet** or **Neumann** boundary condition. Here the scalar function a is such that $0 < a_* \leq a \leq a^*$. We will concentrate on the case of **generalized solutions** in $W^{1,p}(\Omega)$ with $1 < p < \infty$.

Problem 3. Degenerate case. We will finally consider the following problem:

$$-\text{div}\,(\varrho^\alpha \nabla u) + k\frac{u}{\varrho^\beta} = f \quad \text{in } \Omega, \tag{3}$$

with or without boundary condition and where k is a non negative constant and α and β belong to the interval $[0, 1]$.

Keywords: Elliptic problems, Lipschitz and $\mathcal{C}^1$ domains, maximal regularity, traces, fractional and weighted Sobolev spaces

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Time decay for non-Newtonian-micropolar fluids

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Joint work with Felipe W. Cruz (Universidade Federal de Pernambuco, Brazil) and Marko A. Rojas-Medar (Universidad de Tarapacá, Chile).

Abstract

In this contribution, we analyze the long-time behavior of the weak solutions for a model of micropolar fluids of non-Newtonian type in $\mathbb{R}^2$. Time decay estimates are deduced, and the differences with the linear part of the model are studied.

Keywords: Time decay, micropolar fluids, non-newtonian fluids.

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On a compressible MHD flow interacting with thermoelastic structure

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Abstract

We consider an interaction problem between a full compressible electrically conducting fluid and a thermoelastic shell in two-dimensional framework. The shell is modeled by linear thermoelasticity equations, and encompasses a time-dependent domain which is filled with a fluid described by full compressible (non-resistive) magnetohydrodynamic equations. The magnetohydrodynamic flow and the shell are fully coupled, resulting in a fluid-structure interaction problem that involves heat exchange. The existence of weak solutions is established.

This is a joint work with Binkang Huang and Šárka Nečasová.

References

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A remark on the objectivity of stress tensor in mass diffusive compressible fluid model

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Abstract

The mass diffusive model of Svärd [1] and its symmetrized version introduced by Bodnár and Fraunié [2], recently studied in Bodnár and Mácha in [3] is discussed here from the point of view of objectivity of the constitutive part of the stress tensor. The class of Euclidian frame transformations is considered, taking into account the time dependent rotation and translation of the reference frame . The corresponding transformation properties of stress tensor are first derived for the classical Navier-Stokes-Fourier model, and then compared with those of the original (non-symmetric) mass diffusive model and its symmetrized version. It is shown that the mass diffusive models (their stress tensors) are not objective in general and only in specific scenarios may be close to objectivity.

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High and low-Mach number regimes for capillary fluids

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Abstract

The talk will be devoted to the high and low Mach number limit for compressible capillary fluids with density dependent viscosity. We will present recent results concerning high-Mach number flows, weak-strong uniqueness property and dispersion of the acoustic waves in the low-Mach number regime. This is a joint work with Donatella Donatelli (DISIM, L'Aquila) and Lars Eric Hientzsch (KIT, Karlsruhe).

Keywords: Mach number regimes, capillary fluids, dispersion of acoustic waves.

References

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Stokes resolvent with traction boundary conditions

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Abstract

Let Ω denote an open, bounded set in $\mathbb{R}^3$ with connected C^2-boundary $\partial\Omega$, and put $\overline{\Omega}^c := \mathbb{R}^3\backslash\overline{\Omega}$ (exterior domain). Let $\lambda \in \mathbb{C}\backslash(-\infty, 0]$. Consider the Stokes resolvent system

$$-\Delta u + \lambda u + \nabla\pi = F, \ \ \mathrm{div}\, u = 0 \quad \text{in } \overline{\Omega}^c, \tag{1}$$

under traction boundary conditions

$$\sum_{k=1}^{3} (\partial_j u_k + \partial_k u_j - \delta_{jk}\,\pi)\, n_k^{(\Omega)} = B_j \quad \text{on } \partial\Omega \quad \text{for } \ 1 \le j \le 3, \tag{2}$$

where $n^{(\Omega)}$ denotes the outward unit normal to Ω. Let $p \in (1, \infty)$. By $D^{1,p}(\overline{\Omega}^c)$ we denote the space of all functions $\sigma \in W^{1,1}_{loc}(\overline{\Omega}^c)$ such that $\nabla\sigma \in L^p(\overline{\Omega}^c)^3$. Let $\vartheta \in [0, \pi)$. We are interested in solutions $(u, \pi) \in W^{2,p}(\overline{\Omega}^c)^3 \times D^{1,p}(\overline{\Omega}^c)$ of (1), (2), as well as in the estimate

$$|\lambda|\, \|u\|_p + \|u\|_{2,p} + \|\nabla\pi\|_{1,p} \le \mathfrak{C}\, \|F\|_p \tag{3}$$

for $F \in L^p(\overline{\Omega}^c)^3$ and $\lambda \in \mathbb{C}$ with $|\arg\lambda| \le \vartheta, \ |\lambda| \ge \lambda_0$, where $\mathfrak{C}$ and λ_0 are constants only depending on p, ϑ and Ω (estimate uniform with respect to λ with $|\arg\lambda| \le \vartheta, \ |\lambda| \ge \lambda_0$). If inequality (3) is available, problem (1), (2) may be written as an equation in terms of an operator ("Stokes operator") generating an analytic semigroup in a suitable space. This equation, which does not involve the pressure, provides an access to the time-dependent Stokes system supplemented by boundary conditions (2).

Grubb [1] expressed a solution of (1), (2) in terms of a pseudo-differential operator on $\partial\Omega$, but did not make explicit any regularity property or estimate of this solution. Shibata, Shimizu [3], [4] and Shibata [2] addressed problem (1), (2) by reducing it to a boundary value problem in half-space in $\mathbb{R}^3$. They obtained solutions in $W^{2,p}(\overline{\Omega}^c)^3 \times D^{1,p}(\overline{\Omega}^c)$ and proved (3).

Following suggestions by T. Hishida, we showed that problem (1), (2) admits two distinct solution classes in $W^{2,p}(\overline{\Omega}^c)^3 \times D^{1,p}(\overline{\Omega}^c)$, one consisting of functions (u, π) with $\int_{\partial\Omega} u \cdot n^{(\Omega)}\, do_x = 0$ (zero flux of the velocity through $\partial\Omega$), the other one characterized by the relation $\pi|B_R^c \in L^r(B_R^c)$ for some $r \in (1, \infty)$ and some $R > 0$ with $\overline{\Omega} \subset B_R$ (L^r-integrability of the pressure near infinity). The second class exists only if $p > 3/2$, and in the case $p \geq 3$, estimate (3) holds for solutions in this class only if $\operatorname{div} F = 0$ in the sense of distributions. A Stokes operator is associated with both this classes, in the case of the second under the assumption $p > 3/2$, but without the restriction $p < 3$.

There is a one-dimensional subspace of $W^{2,p}(\overline{\Omega}^c)^3 \times D^{1,p}(\overline{\Omega}^c)$ whose nonvanishing elements (u, π) satisfy (1) with $F = 0$ and (2) with $B = 0$, with u not being constant.

Keywords: Stokes resolvent, traction boundary conditions, Stokes operator.

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Viscous flow around an obstacle with oscillating boundary

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Abstract

We consider the flow of an incompressible viscous fluid in a domain with a periodically moving boundary. The problem is reformulated as a time-periodic quasilinear problem in a domain with fixed boundary. Time-periodic maximal L^p-regularity for the linearized problem is established using the theory of operator-valued Fourier multipliers. The existence of solutions to the nonlinear problem in a bounded domain follows by a fixed-point argument under suitable smallness assumptions. For the case of an exterior domain the situation is more involved: If the net motion over one period is non-zero, then one can derive a similar existence result in the framework of homogeneous Sobolev spaces. If the net motion vanishes, existence is shown by combining the maximal-regularity result with pointwise decay estimates that follow from the associated time-periodic fundamental solutions.

Keywords: exterior domain, time-periodic solution, Navier–Stokes equations, moving domain, maximal regularity.

Asymptotic behavior ($t \to +\infty$) of solutions to Navier-Stokes equations under Navier boundary conditions

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Abstract

We study the asymptotic behaviour of the solutions to Navier-Stokes unforced equations under Navier boundary conditions in a wide class of merely Lipschitz domains of physical interest that we call *sectors*. The main motivations come from the celebrated results by Foias-Saut [1] related to the long time behaviour of the solutions to Navier-Stokes equations under Dirichlet conditions.

Here the choice of the boundary conditions requires carefully considering the geometry of the domain Ω, due to the possible lack of the Poincaré inequality in presence of axial symmetries. In non-axially symmetric domains we show the validity of the Foias-Saut result about the limit at infinity of the Dirichlet quotient, in axially symmetric domains we provide two invariants of the flow which completely characterize the motion and we prove that the Foias-Saut result holds for initial data belonging to one of the invariants.

Finally we study the long-time behaviour of the Dirichlet quotient in particular domains, e.g. $\Omega = (0, \pi)^d$ with $d \in \{2, 3\}$, providing further results both from analytical and numerical point of view.

Keywords: Navier boundary conditions, long time behaviour.

References

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Numerical simulation of instabilities as driving processes in the ocean surface

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Abstract

Due to the high resolution of coupled ocean-atmosphere models downscaling up to hundreds meters, a renew of interest is dedicated to basic processes occurring at the sea upper layer including surface and internal waves, mixing processes and coherent structures. From high resolution in situ observations using gliders, floating platforms, HF radars and oceanographic vessels cruises, a review of documented data bases is given. Starting from the Orr Sommerfeld equation as applied to the inflectional instability and depending on initial and

boundary conditions, Kelvin Helmholtz, Holmboe, Ekman and Langmuir secondary flows are especially analysed. In the end, recent investigations are discussed concerning the winter marine deep convection related to Rayleigh Bénard cells and millimetric surface waves as observed in laboratory at the air-sea interface and on the wall of crowns produced by drops impacting the free surface.

Keywords: Ocean surface, Kelvin Hemholtz instabilities, Numerical modelling.

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The steady Navier-Stokes equations in a system of unbounded channels with sources and sinks

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Abstract

The steady motion of a viscous incompressible fluid in a junction of unbounded channels with sources and sinks is modeled through the Navier-Stokes equations under inhomogeneous Dirichlet boundary conditions. Under a general outflow constraint, we prove the existence of a solution with a uniformly bounded Dirichlet integral in every compact subset. The main novelties of our approach are the construction of a flux carrier satisfying a uniform Leray-Hopf inequality in rectangular sections and the proof of some properties of weak solutions to the stationary Euler equations in bounded planar domains, such as the regularity of the extension to the whole plane, of the related Bernoulli pressure and of the stream function. This regularity is used to obtain local Morse-Sard-type information and to generate a solution through the *invading domains* procedure. For small data of the problem, we also prove unique solvability and attainability of Couette-Poiseuille flows at infinity.

Keywords: Steady Navier-Stokes equations, unbounded domain, Leray's problem.

References

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Derivation and analysis of a Stokes-transport system modeling thermoregulation in human skin

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Abstract

We consider a Stokes flow coupled with advective-diffusive transport in an evolving domain with boundary conditions allowing for inflow and outflow. The evolution of the domain is induced by the transport process, leading to a fully coupled problem.
Our approach aims to model the thermal control of blood flow in human skin and the underlying physiological processes. To this end, the model takes into account the temperature-dependent production of biochemical substances, the subsequent dilation of blood vessels, and the resulting changes in convective heat transfer.
The talk will give insights into the main analytical challenges that arise in proving the existence of weak solutions for the system of PDEs. In particular, I will present a fixed point method that allows us to treat the nonlinear coupling.
This is joint work with Maria Neuss-Radu.

Keywords: Stokes equations, heat transport, free boundary problem, coupled problem, vasodilation

Analysis of compressible hydrostatic flows

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Abstract

In this talk we develop the hydrostatic Lagrangian approach to the compressible primitive equations and establish a global, strong well-posedness result for small data under certain assumptions on the pressure law. We also discuss a related atmosphere-ocean model.

This is joint work with Y. Iida, A. Roy and T. Zöchling.

Regularity of very weak time-periodic Poiseuille-type solutions

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Abstract

We present our results concerning the time-periodic very weak solutions of the heat equation with a side condition of the prescribed flux

$$F(t) = \int_\sigma U(x,t)\mathrm{d}x$$

(see [3]). The regularity of very weak time-periodic Poiseuille-type solutions when $F \in W^{\beta,2}(-\pi,\pi)$, $0 \le \beta \le 1$, is investigated. Specifically, we provide an example of a given flux function F that belongs to $L^2(-\pi,\pi)$ but $F \notin W^{\beta,2}(-\pi,\pi)$ for $0 < \beta < 1$. The very weak solution corresponding to such F, has the regularity as stated in the definition (see [1],[2]) and it is not better. Furthermore, we discuss results that suggest a correlation between the improvement of solutions regularity and the increase in regularity of the given function F.

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Existence of strong solutions to a class of compressible non-Newtonian Navier-Stokes equations

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Abstract

We discuss the local-in-time existence of a strong solution to the generalized compressible Navier-Stokes equation for arbitrarily large initial data. The existence of the solution is obtained by the maximal L^p–L^q–regularity theorem for linearized equations which is proven with help of the Weis multiplier theorem. The result, published in [2], can be seen as generalization of the work of Shibata and Enomoto [1] to compressible non-Newtonian fluid.

Keywords: non-Newtonian fluids, the Weis theorem, L^p-theory.

References

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Maximal regularity of Stokes problem with dynamic boundary condition — Hilbert setting

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Abstract

In this contribution we discuss the maximal regularity of weak solutions in time for the evolutionary Stokes problem with dynamic boundary condition. Due to the characterization of R-sectorial operators on Hilbert spaces, the proof reduces to identifying the appropriate functional analytic setting and proving that the corresponding operator is sectorial, i.e., that it generates an analytic semigroup.

This is joint work with Tomáš Bárta and Paige Davis.

Keywords: Stokes problem, dynamic boundary conditions, maximal regularity, analytic semigroup

References

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On wild solutions to the compressible Euler system

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Abstract

In this talk, we survey recent results concerning the existence and properties of so-called wild solutions to the compressible Euler system in two spatial dimensions. These solutions are constructed using the convex integration method developed in this context by De Lellis and Székelyhidi. We introduce the Riemann problem for the compressible Euler system, classify its one-dimensional self-similar solutions, and summarize results related to the uniqueness and non-uniqueness of these solutions within the class of admissible weak solutions. We also discuss existence of wild solutions for regular initial data and the failure of criteria based on maximal dissipation of energy to select physically relevant solutions.

Keywords: compressible Euler system, wild solutions, convex integration, Riemann problem, admissibility

Some controllability results for 1D linearized compressible Navier-Stokes equations

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Abstract

In this talk, I will give a brief overview of the controllability results of Navier-Stokes equations for compressible fluids in one dimension. We will consider the cases where distributed or boundary control(s) act in the density and/or velocity. In each cases, we will see both the existing and new controllability results. Finally, I will conclude my talk with some future directions and open problems.

Keywords: Navier-Stokes equations, compressible fluids, controllability, boundary control.

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Automated design and simulation workflow for an axial blood pump

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Abstract

This talk will present a collaborative effort to develop an automated and extensible workflow for the design and evaluation of an axial blood pumps. The core of the framework is a fully parametric CAD model, which is directly linked to numerical simulations, optimalization and automated report generation. Both the fully parametric CAD model and surrounding Framework was developed in-house.

The workflow is designed to enable systematic performance assessment and shape refinement, while being flexible and extensible. The long term goal is to establish a reproducible and shareable benchmark that can support future extensions, including for example the integration of advanced metrics such as blood damage prediction and thrombus formation.

Keywords: Parametric CAD design, geometrical constructions, ventricular assist device, axial blood pump, incompressible Navier-Stokes, OpenFOAM, optimalization, shape optimalization, automated design workflow, biomedical

Inversely designing boundaries from observed shock fronts in two gas dynamic models

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Abstract

When supersonic flows past wedges and conical obstacles, or the piston expanding into the static gas, there exists a leading shock in the gas. We aim to design the position of obstacles or the trajectory of pistons with the prescribed leading shock and the incoming flow or initial states. By the method of characteristics and asymptotic analysis, under suitable assumptions on the given shock and the given incoming flows or initial states, we globally determine the smooth boundaries and the piece-wise smooth flow field. Moreover, the obtained flow fields admit large velocity variations.

Keywords: Hyperbolic conservation laws, inverse problems, large variation flow field, supersonic flow past obstacles, piston models.

A diffuse interface model of Fluid-Structure Interactions for blood flows and thrombus

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Abstract

This talk concerns a diffuse interface model for the flow of two incompressible viscoelastic fluids in a bounded domain. More specifically, the fluids are assumed to be macroscopically immiscible, but with a small transition region, where the two components are partially mixed. Considering the elasticity of both components, one ends up with a coupled Oldroyd-B/Cahn-Hilliard type system, which describes the behavior of two-phase viscoelastic fluids. In particular, the model describes the interaction between thrombus and blood flows in human arteries. I will present some techniques we employed to prove the existence of weak solutions, which account for the poor compactness of the left Cauchy-Green tensor. Moreover, I will show recent progress on the global strong well-posedness in two dimensions. This talk is partially based on joint work with Dennis Trautwein (Regensburg).

Keywords: fluid-structure interactions, diffuse interface model, blood thrombus, viscoelastic fluid, Navier–Stokes/Cahn–Hilliard.

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On time-periodic solutions to an interaction problem between compressible viscous fluids and viscoelastic beams

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Abstract

We study a nonlinear fluid-structure interaction problem between a "square-root" viscoelastic beam and a compressible viscous fluid. The beam is immersed in the fluid which fills a two-dimensional rectangular domain with periodic boundary conditions in both directions, while both the beam and the fluid are under the effect of time-periodic forces. By using a decoupling approach, at least one time-periodic weak solution to this problem is constructed which has a bounded energy and a fixed prescribed mass. The lack of a priori energy bounds is overcome by a series of estimates based on a careful choice of parameters. The most challenging one is the pressure estimate, which is obtained by utilizing the specific periodic geometry and the Bogovskii operator on a fixed domain that has a uniform constant.

Continuum theories for liquid crystals and their applications

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Abstract

crystals are classical examples of partially ordered materials that combine fluidity with the directionality and ordering of solids. Liquid crystals are anisotropic materials, with direction-dependent physical, optical and rheological properties, making them the working material of choice for a range of opto-electronic devices, e.g., liquid crystal displays, photonics, sensors, photovoltaics etc. In this talk, we review the mathematical theories for three canonical liquid crystal phases: nematic liquid crystals, smectic liquid crystals and cholesteric liquid crystals. We focus on Landau-de Gennes type theories for these phases and describe the essential mathematical frameworks - the liquid crystal order parameters, the free energies, the governing systems of partial differential equations and their solution landscapes. We conclude with some case studies that illustrate the applications of these mathematical frameworks to prototype devices and real-life experiments. All collaborations will be acknowledged throughout the talk.

Keywords: liquid crystals, Landau-de Gennes theory, continuum mechanics, non-linear partial differential equations, variational methods etc.

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Weak solutions for fluid-structure interaction problems with three-dimensional structural displacements

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Abstract

This talk addresses a nonlinear fluid-structure interaction (FSI) problem involving the Navier-Stokes equations, which govern the flow of an incompressible, viscous fluid in a three-dimensional domain, coupled with the motion of a thin viscoelastic lateral wall. The wall's dynamics are modeled by a two-dimensional plate equation with fractional damping, allowing for displacements in all three spatial directions. The system is nonlinearly coupled through kinematic and dynamic conditions imposed on the moving fluid-structure interface, whose position evolves and is not known a priori.

We establish three key results for this class of FSI problems, focusing on cases with vector displacements of thin structures. First, we prove a hidden spatial regularity in the structural displacement, ensuring the absence of self-contact within a finite time interval. Second, we show temporal regularity for the fluid velocity and structural displacement, leading to a novel compactness result for three-dimensional structural dynamics. Finally, leveraging these findings, we construct a proof of local-in-time existence for weak solutions to the FSI problem using a Lie operator splitting method and time discretization.

These results contribute to the mathematical understanding of fluid-structure interaction problems with three-dimensional structural displacements, particularly in terms of solution regularity and well-posedness.

This is joint work with Sunčica Čanić and Krutika Tawri.

Keywords: fluid-structure interaction, weak solution, Navier-Stokes equations.

Effective interface laws for fluid flow and solute transport through thin reactive porous layers

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Abstract

We consider a coupled model for fluid flow and transport in a domain consisting of two bulk regions separated by a thin porous layer. The thickness of the layer is of order ε and the microscopic structure of the layer is periodic in the tangential direction also with period ε. The fluid flow is described by an instationary Stokes system, properly scaled in the fluid part of the thin layer. The evolution of the solute concentrations is described by a reaction-diffusion-advection equation in the fluid part of the domain and a diffusion equation (allowing different scaling in the diffusion coefficients) in the solid part of the layer. At the microscopic fluid-solid interface inside the layer nonlinear reactions take place. This system is rigorously homogenized in the limit $\varepsilon \to 0$, based on weak and strong (two-scale) compactness results for the solutions. These are based on new embedding inequalities for thin perforated layers including coupling to bulk domains. In the limit, effective interface laws for flow and transport are derived at the interface separating the two bulk regions. These interface laws enable effective mass transport through the membrane, which is also an important feature from an application point of view. This is a joint work with Markus Gahn (Universität Heidelberg).

Keywords: Stokes equations, Reaction-diffusion-advection equations, Effective interface laws, Homogenization, Two-scale convergence.

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A necessary condition for the vanishing singularity in a suitable weak solution of the MHD equations

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Abstract

We explain what we mean by the vanishing singularity. Then we show that if $(\mathbf{x}_0, T)$ is a vanishing singularity in a suitable weak solution $(\mathbf{u}, \mathbf{b})$ of the MHD equations in a 3D domain Ω, where $\mathbf{u}$ is the velocity and $\mathbf{b}$ is the magnetic field, then

$$\lim_{t\to T-} \left(\|\mathbf{u}(\,.\,,t)\|_{\xi;\,B_R(\mathbf{x}_0)} + \|\mathbf{b}(\,.\,,t)\|_{\xi;\,B_R(\mathbf{x}_0)}\right)(T-t)^{\frac{\xi-3}{2\xi}} \;=\; \infty$$

for any $\xi \in [3,6]$ and $R > 0$.

Keywords: MHD equations, weak solution, regularity

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Darcy's law for inhomogeneous incompressible flows

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Abstract

We investigate the homogenization of a density-dependent incompressible fluid confined to a bounded three-dimensional perforated domain. Assuming that the perforations are large enough, when letting the number of inclusions go to infinity by shrinking them at the same time, we will show that the limiting (effective) equation is given by a density-dependent incompressible Darcy law. Additionally, we will give convergence rates, as well as the existence of a strong solution to the limiting system. This is joint work with Danica Basarić (Politecnico di Milano) and Jiaojiao Pan (Nanjing University).

Keywords: Homogenization; Non-homogeneous; Navier-Stokes system; Darcy's law

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Non-stationary Navier-Stokes equations in $2D$ power cusp domain

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Abstract

The initial boundary value problem for the non-stationary Navier-Stokes equations is studied in 2D bounded domain with a power cusp singular point O on the boundary. We consider the case where the boundary value has a nonzero flux over the boundary. In this case there is a source/sink in O and the solution necessary has infinite energy integral. The asymptotic decomposition of the solution near the singular point is constructed and justified, i.e., existence of the solution which is represented as the sum of the constructed asymptotic expansion and a term with finite energy norm is proved. Moreover, it is proved that the solution represented in this form is unique.

Keywords: Non-stationary Navier-Stokes problem, power cusp domain, singular solutions, asymptotic expansion.

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Numerical modeling and simulation of viscoelastic fluid flows: Challenges and approaches

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Abstract

This presentation addresses the modeling and numerical simulation of viscoelastic fluid flows, with a particular focus on the Oldroyd-B model. We begin by motivating the study of viscoelastic fluids, highlighting their relevance in polymer processing, hemodynamics, and industrial applications. The mathematical formulation is presented, including the governing equations and dimensionless parameters, with emphasis on the High Weissenberg Number Problem that arises in regimes of strong elasticity. We then discuss numerical challenges and stabilization techniques, such as artificial stress diffusion, essential for ensuring robust and accurate simulations. Finite element approximations are introduced as a framework for numerical discretization, and the interplay between Eulerian and Lagrangian perspectives is discussed. Finally, simulation results for two-dimensional channel flows are presented, illustrating both steady-state solutions and the limitations of the models under high elasticity. Concluding remarks highlight the potential for further research and improvement in computational modeling of viscoelastic fluids.

Keywords: viscoelastic fluids, Oldroyd-B model, numerical stabilization stress diffusion. .

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Weak solutions for compressible viscoelastic fluid models in three space dimensions

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Abstract

We discuss global in time existence of weak solutions to compressible visco-elastic fluid models in three space dimensions. The first result concerns the situation with corrotational derivative in the extra stress tensor. Then, assuming additionally that the extra stress tensor has a particularly simple structure, the existence of weak solutions can be shown even in the situation when the stress diffusion is neglected which is often the case in applications.

The second result concerns Oldroyd-B type of model. It is known that in three space dimensions the Newtonian structure for the viscous part of the stress tensor is not enough to ensure the existence of weak solutions for arbitrarily large data. However, assuming the stress tensor of the power-law type it is possible to close the estimates and construct solutions provided the extra stress diffusion is present and the model of the viscous stress tensor provides bounded velocity divergence.

Keywords: compressible visco-elastic fluid, weak solution, large data.

Incompressible heat-conducting fluid with large flux: long time solutions.

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Abstract

We consider the incompressible Navier-Stokes equations coupled with the heat equation in a cylindrical domain Ω parallel to the x_3-axis. On the lateral part of the cylinder we assume the slip boundary conditions for velocity v, so there is no flow through the side wall, but there is flow through the top and bottom, where we have possibly large inflow and outflow. Moreover, we assume homogeneous Neumann boundary condition for temperature θ, meaning thermal insulation. With smallness of the x_3-derivative of initial velocity, initial temperature and the external force field and smallness of derivatives of the inflow and outflow, we prove long time estimates for v and θ in Sobolev spaces $W_2^{2,1}(\Omega^t), t \leq T$ with any finite T. Next, by Leray-Schauder fixed point theorem we show the existence of solutions to the problem in space $W_2^{2,1}(\Omega^t), t \leq T$. Thanks to the special geometry of domain Ω we are able to prove the existence of solutions with large magnitudes of initial velocity, temperature and inflow-outflow functions.

Keywords: incompressible Navier-Stokes equations, heat equation, inflow-outflow.

A conditional no-contact result of compressible fluids and elastic plates in 2D

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Abstract

We consider the interaction of a compressible fluid with a flexible plate in two space dimensions. The fluid is described by the Navier-Stokes equations in a domain that is changing in accordance with the motion of the structure. The displacement of the latter evolves according to a beam equation. Both are coupled through kinematic boundary conditions and the balance of forces. We prove that for any weak solution to the coupled system, which satisfies certain additional regularity requirements, no contact occurs between the elastic wall and the bottom of the fluid cavity.

This is a joint work with Dominic Breit.

Keywords: Compressible fluids Elastic plates Fluid-structure interaction Contact problem.

Computational modeling of blood rheology in cerebral aneurysm flow dynamics

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Abstract

Cerebral aneurysms are pathological dilations of intracranial arteries that present serious health risks due to potential rupture and subarachnoid hemorrhage. Despite advances in imaging and treatment, the mechanisms driving aneurysm development and rupture remain unclear. Image-based simulations are valuable tools in this research, but often oversimplify blood rheology. While many models treat blood as a Newtonian fluid, it exhibits shear-thinning non-Newtonian behavior that can significantly influence flow predictions, especially in low-shear regions like aneurysm domes.

This talk explores how blood rheology affects CFD outcomes in image-based aneurysm models. Comparing Newtonian and non-Newtonian approaches, we find significant differences in wall shear stress, oscillatory shear index, and flow structures. These results highlight the need for realistic blood modeling to enhance the accuracy and clinical utility of simulations.

Keywords: image-based cerebral aneurysms, non-Newtonian shear-thinning models, hemodynamics.

On the stationary Navier-Stokes equations in distorted channels and pipes under the DDN boundary condition

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Abstract

We consider the steady motion of a viscous incompressible fluid in distorted channels or pipes, of finite length, modeled through the Navier-Stokes equations with mixed boundary conditions: the inflow is given by an arbitrary member of a Lions-Magenes class, and the fluid motion is subject to a directional do-nothing boundary condition (DDN) on the outlet, together with the standard no-slip assumption on the remaining walls of the domain. Existence of a weak solution to such Navier-Stokes system is proved without any restriction on the data, that is, inlet velocity and external force. Under a suitable smallness assumption on the data, we also prove the unique solvability of the boundary-value problem. This is joint work with Alessio Falocchi (Dipartimento di Matematica, Politecnico di Milano, Italy) and Gianmarco Sperone (Facultad de Matemáticas, Pontificia Universidad Católica de Chile, Santiago).

Keywords: incompressible fluids, mixed boundary conditions, channels, pipes.

Boundary value problems for the Stokes equations in an infinite layer

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Abstract

This contribution is about a long term cooperation with Konstantin Pileckas and Sebastian Rauchhaus and it contains some results about problems that were solved but also one problem is presented which we could not solve up to now. Already 1999 Nazarov and Pileckas [1] published a paper about asymptotic behavior of solutions to the stationary Stokes Dirichlet Problem in a layer like domain. Apart from the explicit expressions for the asymptotics this paper also included precise estimates for the remainder in appropriate weighted Sobolev spaces. In this context it is quite natural to think about generalizations like what about

- the nonlinear case?
- existence and uniqueness of solutions in this special form?
- instationary problems?
- general boundary conditions?

In this lecture I will focus on the linear Stokes problem up to the point where we could not come around an obstacle.

Keywords: Stokes problems, layer like domain, asymptotics of solutions

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Energy conservation for compressible fluid systems with Korteweg stress tensors

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Abstract

Energy conservation is an important issue in Onsager's conjecture. In this talk, we consider the weak solutions of compressible quantum Euler system and quantum Navier-Stokes system under what regularity conditions conserve the energy. Based on the work of Bresch et al. (Arch. Rational Mech. Anal. 223: 975-1025, 2019) and Feireisl et al. (Arch. Ration. Mech. Anal. 223: 1375-1395, 2017), we introduce the drift velocity and the effective velocity to write the two quantum fluid systems and obtain the corresponding augmented systems as the compressible Navier-Stokes system with density dependent viscosity, then prove the energy conservation for the augmented system, which eliminate the third order dispersive term. We find some new observations and phenomena, which is different from the previous results.

Keywords: Quantum fluid systems; Energy conservation; Onsager's conjecture

Contact in fluid-plate interaction: formation and detachment

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Abstract

In this talk, I will present a very recent result on contact problem for the interaction between an elastic plate and a compressible viscous fluid located between the plate and a rigid bottom $z = 0$. First, by utilizing the vertical fluid dissipation, a new estimate is obtained $\ln \eta(t) \in L^1$ for any $t > 0$ provided that $\ln \eta_0 \in L^1$, ensuring that additional contact can form only on a set of a measure zero. Then, by utilizing the expanding capability of compressible fluid pressure, it is shown that all contact has to detach in finite time provided that the source force is not pushing down too much. Finally, it is shown that contact at any point can be detached in any given time with a strong enough source force localized around that point which is pulling the plate up. This is the first result where detachment of contact is proven.

Keywords: fluid-structure interaction, contact problem, post-contact dynamics, compressible Navier-Stokes.

Mathematical and numerical model of Human Phonation

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Abstract

This talk will address mathematical and numerical model of very complex process represented by human phonation. It involves interaction of three different physical fields – the deformation of the vocal folds (elastic body), the complex fluid flow and the acoustics together with their mutual couplings, usually referred as the fluid-structure-acoustic interaction (FSAI). Further, mutual contact of both vocal folds regularly appears during healthy phonation increasing the difficulty of the considered problem.

First, the modelling approach leading to decoupling FSI problem including contact and the aeroacoustic problem will be introduced. The two-dimensional FSI problem is modelled with the aid of linear elastic problem coupled to the incompressible Navier-Stokes equations in the arbitrary Lagrangian-Eulerian form. The contact treatment suitable for the chosen finite element framework is briefly introduced. Further, the aeroacoustic problem is introduced. Two different mathematical descriptions – the Lighthill acoustic analogy and the Aeroacoustic Wave Equation, are given and used. In the end, the parameter sensitivity study is shown and the resulting frequency spectra for phonation of vowel [u:] are presented.

Keywords: fluid-structure-acoustic interaction, human phonation, Navier-Stokes equations, finite element method, aeroacoustics.

On Leray's structure theorem

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Abstract

Let $\Omega \subseteq \mathbb{R}^3$ be a bounded domain with $\partial\Omega \in C^\infty$, and let $0 < T \leq \infty$. In $[0,T) \times \Omega$ we consider a general weak solution of the Navier-Stokes equations

$$u_t - \Delta u + u \cdot \nabla u + \nabla p = f, \qquad \nabla \cdot u = 0, \qquad u|_{\partial\Omega} = 0, \qquad u|_{t=0} = u_0,$$

where $u_0 \in W^{1,2}_{0,\sigma}(\Omega)$ and $f = \operatorname{div} F$, $F \in C^\infty_0([0,T); C^\infty(\overline{\Omega}))$, are given data. Our main result concerns Leray's structure theorem, see [2, p. 244]. In particular, for the special case $F = 0$, $T = \infty$, and u satisfying the strong energy inequality

$$\frac{1}{2}\|u(t)\|_2^2 + \int_{t_0}^t \|\nabla u\|_2^2 \, d\tau \leq \frac{1}{2}\|u(t_0)\|_2^2$$

for almost all $t_0 \in [0,T)$ and all $t \in [t_0,T)$, it is known [1, pp. 57] that there exists an open local in time regularity region $R \subseteq (0,T)$ such that $u \in C^\infty(R; C^\infty(\overline{\Omega}))$. We extend this result to several directions: Instead of $F = 0$, $T = \infty$ we allow $F \neq 0$, $0 < T \leq \infty$ as above, and we admit a general weak solution u in $[0,T) \times \Omega$ in the usual sense, without assuming the strong energy inequality.

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Uniqueness of weak solutions in hyperbolic-parabolic systems with applications in poro-elasticity

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Abstract

We focus on the uniqueness of weak solutions to hyperbolic-parabolic PDE systems. Relevant challenges are motivated by a linear poro-elastic filtration system [1] of recent interest, coupling a 3D system of elasticity and a 3D free flow. The construction of finite-energy weak solutions can be done in several ways [1, 4], however, as is typical, weak solutions do not reside in the test class [5]. As such, an additional argument is needed for uniqueness. The challenge in obtaining a traditional energy estimate for weak solutions comes through the hyperbolic component. Interface coupling brings about regularity loss in the dynamics, as well as presenting ill-defined traces. These challenges can be circumvented via semigroup methods, but those only provide well-posedness for strong and mild solutions.

To obtain energy estimates for arbitrary weak solutions—and thus continuous dependence and uniqueness—we present two different approaches, encompassing several parameter regimes for a Biot-Stokes model [2]. For a certain degenerate case, we utilize a "hyperbolic regularization" [5], adapting a classical wave equation argument [7] in more general spaces that account for boundary coupling. In the non-degenerate (semigroup) cases, we connect finite energy weak solutions to a particular adjoint notion of weak solutions appearing in [3]. The latter are unique whenever there is an underlying semigroup, so we obtain uniqueness in the former sense through identification. Time permitting, we discuss recent results [6] complementing the classic result in [3] that provide a general tool for obtaining weak well-posedness.

Keywords: 35D30, 35A02, 74F10, 47D06

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Singular limit - from compressible to incompressible, MHD with non-conservative boundary conditions

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Abstract

The aim of the talk will be to present some theory on singular limits in thermodynamics of viscous fluids and how to rigorously obtain incompressible models from compressible ones in certain regimes. We consider a general compressible, viscous, heat and magnetically conducting fluid described by the compressible Naiver–Stokes–Fourier system coupled with induction equation. In particular, we do not assume conservative boundary conditions for the temperature and allow heating or cooling on the surface of the domain. We are interested in the mathematical analysis when the Mach, Froude, and Alfvén numbers are small, converging to zero at a specific rate. We give a rigorous mathematical justification that in the limit, in case of low stratification, one obtains a modified Oberbeck–Boussinesq–MHD system with a non-local term or a non-local boundary condition for the temperature deviation. Choosing a domain confined between parallel plates, one finds also that the flow is horizontal, and the magnetic field is perpendicular to it. The proof is based on the analysis of weak solutions to a primitive system and the relative entropy method. This is a recent joint work with Florian Oschmann and Piotr Gwiazda.

Keywords: Naiver–Stokes–Fourier system, MHD, low Mach number, low Alfvén number, Oberbeck–Boussinesq–MHD, relative entropy method.

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